Direct Modeling of Inductor Saturation Behavior in a SPICE-Like Transient Analysis

P. Winkler and W. Günther

***Abstract*—In this paper, we demonstrate how the saturation behavior of an inductor can be directly inserted as a software function into the mathematical description of a circuit and included in a SPICE-like numerical simulation. Within the numerical computation of the circuit, the inductance value changes in dependence on the actual inductor current, following the real-life behavior of the choke. The procedure shown provides more exact and realistic simulation results then assuming the inductance to be a constant value, which is the common way in SPICE programs. Based on an example choke, we show how any saturation curve, derived from measurement data or core manufacturer's information, can be inserted into a computational model of the inductance. This is a significant advantage over the possibilities to model inductor saturation, which different SPICE programs are offering, what is also shown in the paper.**

On the application of a boost converter, the impact of the consideration of the saturation on the simulation result is presented and compared with a simulation with a constant inductor.

***Index Terms*—Circuit modeling, inductor saturation, SPICE, transient simulation.**

I. Introduction

Time space (transient) simulation of electric circuits is a common tool for their development. Inductive components, like chokes, are a main component in many circuits. Also, in modern SPICE versions (SPICE—simulation program with integrated circuit emphasis), inductors can be hardly modeled as what they are—a current-dependent device. Besides some limited exceptions, presented in Section II, the software only accepts a constant value. The assumption that choke inductance is a constant value is correct only if the coke is operating far below its saturation point, at which the inductor current and the magnetic field forced by it will decrease the permeability of the soft magnetic core and so the inductance.

Especially for widely used powder core chokes, this assumption can be problematic and is far from reality. These inductors do not have a sharp saturation point, only a soft saturation behavior, and change their inductance continuously in their normal operation point dependent on the current, as presented in Section III and Section VII.

In a numerical simulation, like a transient SPICE analysis, it could be a fast-forward solution to model the inductance as a current-dependent device. For this reason, the constant inductance will be replaced by a function of its own current, as we demonstrate in this paper. The mathematical description used in this function can be derived by datasheet information or measurement data. It accepts any mathematical operation, which is of great interest. The user is free to describe the saturation by the function, which fits the best, and to include the real behavior of his/her device in its computation.

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II. Related Work

There exist different approaches to model the saturation behavior of an inductor in SPICE. Some are already implemented in different circuit simulators, whereas others are methods of circuit simulation users trying to overcome the limitation of the software. Such solutions are often limited to really specific cases and are hardly usable in general.

A common approach used, for example, in SPICE 2G6 is the utilization of the POLY keyword, as presented by [1] or [2], which is close to the method presented in this paper. In this case, the saturation of the inductance is described by an n-dimensional polynom ($n \leq 20$), as presented in Equation 1:

$$L(I) = L_0 + A_1 \cdot I + A_2 \cdot I^2 + \ldots + A_n \cdot I^n \tag{1}$$

In practice, often just the first three or four summands of this equation are enough to model the saturation curve in some range with the required accuracy. The disadvantage of this method is the boundedness of the equation to be a polynomial function, which is often not available or requires additional curve-fitting effort. The method presented in this paper also describes the inductor as a function of its own current but will accept any mathematical formula.

In the free software, LT-SPICE, the modeling of the inductor saturation is possible in two different ways [3].

The first method uses the flux statement. This method accepts any mathematical formula, which describes the dependency of the total (coupled) magnetic flux in the inductor as a function of the inductor current (indicated by the keyword "x"). Using this formula, the software can compute the transient current, its derivative, and the inductor voltage. The drawback of this method is that the user has to provide the mathematical description of the inductor flux. To get the flux dependent on the inductor current, the saturation curve of the inductance L(I), may it be a mathematical formula or measurement points, has to be integrated, which is difficult and in some cases impossible (e.g., if the saturation is calculated as presented in Equation 6 or 7).

The second way to model the saturation in LT-SPICE is to insert the saturation and remnant flux density and the coercive force of the core material, as well as the number of turns of the choke and the geometrical data of its core (cross section, magnetic path length, and gap size), into the model as predefined parameters. This model is also able to consider hysteresis but needs exact knowledge about the core material

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as well as the choke design. It cannot deal with measurement data and is unusable in the case of a powder core inductor. Powder core chokes, in contrast to a choke based on a gapped ferrite core, have a soft saturation behavior and no measurable air gap, as presented in Section III.

Users tried to overcome the limitations of their SPICE tools and looked for other solutions to implement the saturation in their computer model. One approach is to consider a second inductor winding for biasing, as presented in [4]. This method uses a SPICE equivalent circuit model to describe a partial saturated inductor core by using a gyrator. This saturation model is quite complicated and requires too much modeling and computational effort to be usable in practice.

A more usable technique is presented in [5]. It models the saturation by an imaginary transformer and its reflected impedance, using voltage-controlled voltage and current sources. It states to be able to model measured saturation curves as well as powder cores (KoolMµ saturation curves, which can be derived by the manufacturers' datasheets. It seems to give quite realistic results but also requires a lot of modeling effort to describe the inductor by many additional imaginary devices, which is what makes it abstract and difficult to handle.

III. Saturation Behavior: Powder vs. Gapped Cores

In this section, we present the difference between the saturation curve of an inductor based on a gapped ferrite core and a powder core inductor. The modeling of the saturation curve, as presented in this paper, is able to consider any saturation behavior. It can use measurement data (independent of any knowledge about the choke design, e.g., number of turns, core material, and size) and a mathematical formula, which is always available in the case of a powder core inductor, if the core material and size, as well as the number of turns, are known.

Powder cores consist of very small particles of soft magnetic metal, mechanically pressed to a core but electrically separated by an insulating binder material. The binder is also not magnetic and forms a "distributed air gap" within the core. The different density of the soft magnetic material within a core results in different permeabilities in the range μ=2...550. Ferrite cores consist of sintered material. Their permeability is mostly between μ=1000...12000. In order to store energy and prevent the core from driving into saturation even at low currents, ferrite cores have to be gapped to get their effective permeability down. Due to the different structure of the air gap, the saturation behavior of powder core inductors is much different compared with a choke with a gapped core. Both cases are compared in Fig. 1, where the saturation curve of a powder material with a permeability of 60 is presented, as well as the saturation of a gapped ferrite with the same effective permeability [6]. The figure clearly presents the difference between the soft saturation behavior of a powder core and the sharp saturation of a gapped ferrite.

IV. SPICE Transient Simulation Procedure

In this section, we provide a very short overview of how SPICE software works in the case of a transient simulation and how the inductor saturation curve can be easily inserted into the software.

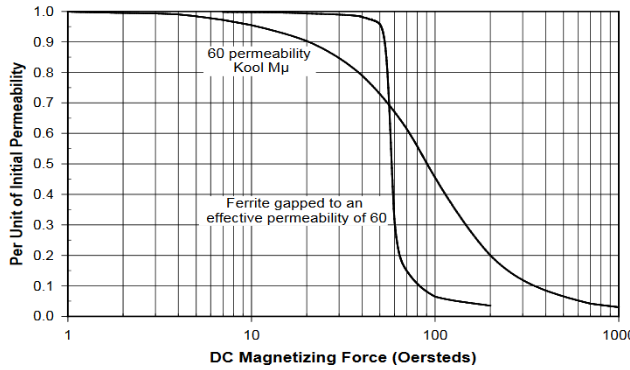

Fig. 1. Saturation curve of a gapped ferrite and a powder core with the same effective permeability [6].

For a transient analysis, all different SPICE tools, based on the same kernel, generate an ordinary differential equation system (ODE system) or a differential-algebraic system of equations (DAE system) from the netlist of the circuit using the modified nodal analysis method [7], [8]. To solve an ODE system or DAE system, different SPICE versions use numerical solvers based on the Gear (also known as the Backward Euler method) or Trapezoidal Rule Algorithm (or a combination of them) to compute the signal curves from one timestep to the next. The two basic procedures are presented in Fig. 2 and explained in detail by [7] or [9]. Similar numerical solvers are available in the MATLAB or Octave software as ODE solvers.

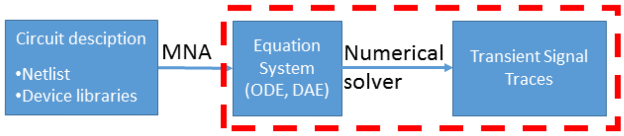

Fig. 2. Transient circuit simulation within a SPICE software.

In this paper, we show how the inductor saturation behavior can be inserted into the mathematical model (equation system) of the circuit and pass the numerical solver to get the time–space behavior of the circuit.

V. Circuit Differential Equation System

If the simulated circuit contains energy storage elements, like capacitors or chokes, the equation system will be an ODE or DAE equation system in case of a transient analysis. The signals of these devices, i.e., the voltages of capacitors or currents through inductors, which are steady (cannot jump), form the state variables of the system. The mathematical (state space) model of the circuit can be expressed as a set (vector) of differential equations, as presented in the following equation:

$$\frac{\partial \vec{x}}{\partial t} = \dot{\vec{x}} = f(t, \vec{x}) \qquad \vec{x}(t=0) = \vec{x_0} \tag{2}$$

$\vec{x_0}$ is a vector consisting of the initial values of the state variables at the start of the transient analysis. Unless otherwise defined, they are all zero by default in most SPICE programs.

VI. Inserting Inductor Saturation Behavior into the Equation System

As the equation system (Equation 2) shows that the derivative of the state variables is dependent on their actual value, it is not difficult to insert the inductance dependency on its own current into this mathematical problem. A current trough an inductor is always a state variable.
If the saturation of the inductor is described as a function

$$L = h(i_L) \tag{3}$$

and we exclude the inductor current from the other state variables, the equation system is presented as follows:

$$\begin{pmatrix} \dot{\vec{x}} \\ \dot{i_L} \end{pmatrix} = f\big(t, \vec{x}, i_L, h(i_L)\big) \qquad \begin{pmatrix} \vec{x}(t=0) = \vec{x_0} \\ i_L(t=0) = i_{L,0} \end{pmatrix} \tag{4}$$

Equation 4 can be written as Equation 2 if we imply i_L into the vector of the other state variables $\vec{x}$.

VII. Modeling Saturation Behavior as a Current-Dependent Inductance in a Numerical Simulation

In this section, we show the transient simulation of a current-dependent inductor in a very simple test circuit, which appears as follows:

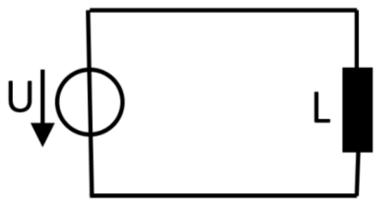

Fig. 3. Test circuit for a transient simulation of saturation.

The circuit consists of a constant DC voltage source and an inductor. The circuit has just one state variable, which is the inductor current. The differential equation to solve is presented as follows:

$$\dot{i_L} = U/L \tag{5}$$

If we consider the voltage source to be a constant DC value and L to be constant, the resulting current will be linearly increasing over time. If we simulate the inductance with its saturation, the current will increase faster, as induction drops at higher currents. In Fig. 4, the inductor current curve is presented for the following three cases:

- inductance is constant $L = 292\ \mu H$
- inductor saturation is modeled as a continuous function $L(i_L)$
- inductor saturation is modeled by a stepwise linear function based on 11 points of the saturation curve, which is derived from measurement data (see Fig. 6)

The voltage source is simulated with a constant value of $U = 10\ V$, the initial value of the current is $i_{L,0} = 0A$, and the simulation time stops at 100 µs.

It can be seen that the modeled saturation behavior leads to the expected results. Some small deviations can be seen between the two different functions for the saturation.

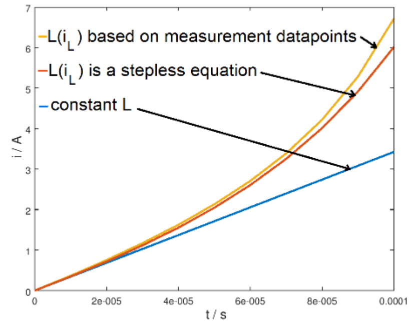

Fig. 4. Inductor current traces modeled with and without saturation.

Fig. 5. Inductor: Core 77310-A7 (KoolMµ material), *N*=57 turns.

A. *Modeling Saturation Behavior as Continuous Function Based on Core Supplier Information*

The equation used to simulate the saturation of the inductance is based on the formula given by the core supplier [10]. The function provided by the core manufacturer shows the drop of the permeability to its initial value (no applied magnetic field) in dependence on the nominal magnetic field strength H. Knowing the number of turns N of the choke and the effective (mean) magnetic path length l_{eff}, as well as the A_L value of the core, this function can be turned into the saturation formula $L(i_L)$, as presented in Equation 6.

$$\frac{\mu}{\mu_i} = \frac{1}{a + b \cdot H^c} \xrightarrow{A_L\ \ N\ \ l_{eff}} L(i_L) = \frac{L_0}{a + b \cdot \left(\frac{N \cdot i_L}{l_{eff}}\right)^c} \tag{6}$$

The parameter of a, b, and c of the core material, as well as the parameter of the core, are provided by its manufacturer.
In the case of Fig. 4, the saturation behavior is formulated for the following choke, which is also presented in Fig. 5:

- core = 77310-A7 (Magnetics KoolMµ material with an initial permeability of μ=125) with an effective magnetic path length of 5.67 cm and A_L= 90 µH
- Number of turns is N = 57

The result is presented in Equation 7 and the graph of the function in Fig. 6.

$$L(i_L) = \frac{292\ \mu H}{1 + 6.345e - 3 \cdot (i_L \cdot 10.053/A)^{1.462}} \tag{7}$$

This formula is inserted into the Octave software by the function L, as presented in the following code:

```
function L_s = L(i)
L_s = 292e − 6/(1 + 6.345 e − 3*( i*10.053)^1.462);
end
```

Equation 5, which is the function describing the state variable signal of the circuit, is inserted as follows:

```
function idot = U_Lcirc(t, i)
U = 10;
Idot = U/L(i);
end
```

It is called by the numerical ode45 solver with the following statement, which also provides the time range to simulate (input 2) and the initial value (i0 – input 3) to start the computation.

```
i0 = 0;
[T2, D2] = ode45(@U_L_circ,[0:1e-5:1e-4],i0);
```

The numerical integration leads to the orange trace presented in Fig. 4.

It is important to consider the function to work correctly only if the drop of the inductance due to saturation is above 30% of its initial value, as stated by the manufacturer [10]. If this is not the case, it may be a better solution to provide the saturation curve as a stepwise linear function based on measurements, as presented in the following subsection.

B. Modeling Saturation Behavior as Stepwise Linear Interpolation of Measurement Points

If the saturation behavior of a choke cannot be described by a mathematical function, it can be inserted into the transient circuit model using a number of measurement points. This is very helpful if information about the core (e.g., material) or the choke (e.g., number of turns) is not available. Figure 6 presents the saturation curve of the choke described above (core: Magnetics 77310-A7, number of turns N=57) calculated using the information of the manufacturer and measured on a sample.

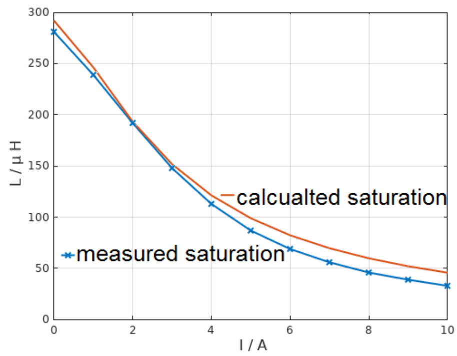

Fig. 6. Calculated and measured saturation – Core 77310-A7, N= 57 turns.

The 11 measured data points $(L(i_n), i_n)$, presented in Fig. 6 as crosses on the blue line, can be transformed into a stepwise linear function $L(i_L)$ (as indicated by the blue trace) and inserted into the program code instead of the function using the core data of the manufacturer. Figure 4 shows that the simulation results of both models are quite similar.

The usage of measured curves within the computational model is of course independent of the core material and is a very practical solution.

VIII. Application Example: Boost Converter Circuit

A. Circuit Description

After presenting how the saturation behavior of an inductor is inserted into the mathematical model of a very simple circuit, we show it in a slightly more complex circuit. The circuit is a boost converter, as presented in Fig. 7.

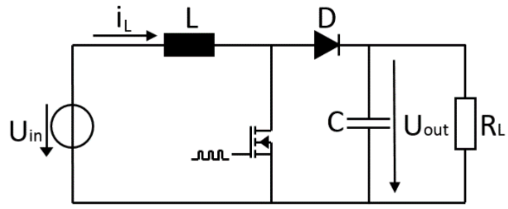

Fig. 7. Boost converter circuit.

It is powered by an input voltage of $U_{in} = 5V$ and controlled by a pulse-width modulation (PWM) signal with a frequency of $f = 31kH$. Its load is $R_L = 30\ \Omega$, and its output capacitance is $C = 20\ \mu F$. For a duty cycle of the PWM signal between D=0.6...0.8, the circuit has an output voltage of $u_{out} = 11.7 \ldots 24.1V$. The circuit has two state variables: the current through the inductor and the (output) voltage of the capacitor. To simplify (as we are not focusing on the semiconductors), we assume the following:

- The transistor is an ideal switch, which is closed if the PWM signal is high and open if the signal is low.
- The diode has a threshold voltage of 0.8 V. At this voltage, the diode becomes an ideal conductor for positive currents; otherwise, the diode blocks the current.

These assumptions lead to the two circuits for the different levels of the PWM signal:

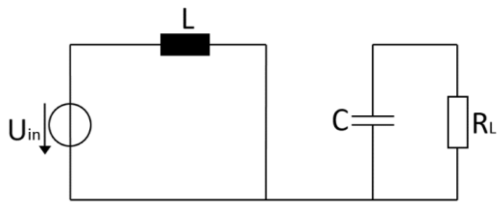

Fig. 8. Boost converter circuit in the state of high gate (PWM) signal.

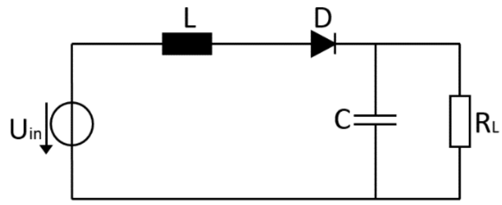

Fig. 9. Boost converter circuit in the state of low gate (PWM) signal.

B. Boost Converter Differential Equation Model

Depending on the state of the PWM signal (Fig. 8, Fig. 9), the differential equation system of the circuit appears as exactly stated in Equations 8, 9, and 10.

$$\dot{\vec{x}} = \begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} \dot{i_L} \\ \dot{u_C} \end{pmatrix} \tag{8}$$

$$\dot{i_L} = \frac{1}{L(i_L)} \begin{pmatrix} U_{in} & for & pwm = 1 \\ U_{in} - u_{out} - 0.8V & for & pwm = 0 \wedge i_L > 0 \\ 0 & for & pwm = 0 \wedge i_L < 0 \end{pmatrix} \tag{9}$$

$$\dot{u_{out}} = \frac{1}{C} \begin{pmatrix} -u_{out}/R & for & pwm = 1 \\ i_L - u_{out}/R & for & pwm = 0 \end{pmatrix} \tag{10}$$

C. Circuit Transient Computation

For the function of the saturation behavior $L(i_L)$, we use exactly the same as shown in the section before, as we consider the same choke (core 77310-A7, number of turns N = 57). The mathematical description of the circuit prepared as an Octave function, which is solvable using the ODE solver of the software, is presented as follows:

```
function dx=Boost ( t , x )
R=30; Ue=5; C=20e-6;
S= Pulssignal ( t ) ;
if S==1
dx1=Ue / L( x ( 1 ) ) ;
dx2=-x ( 2 ) / ( C*R) ;
else
dx1 =(Ue-x ( 2 ) -0 . 8 ) /L( x ( 1 ) ;
dx2 =( x (1)-x ( 2 ) /R) /C;
if ( ( x(1)<=0)&&(dx1 <0))
dx1 =0;
dx2=-x ( 2 ) / ( C*R) ;
end
end
dx =[ dx1 dx2 ] ;
end
```

Note that due to numerical reasons (in the simulation), i_D can drop below 0 (minimal, e.g., $i_D = 11e - 13A$) in a single timestep. So the diode is specified to block under the following conditions:

- transistor is open ($S \neq 1$), and so $i_L = i_D$ and
- the current through the diode is equal or below 0, and its time derivative is negative $(x(1) <= 0)\&\&(dx1 < 0)$.

Describing the diode current in this way enables its value to increase again if it falls below 0 in some timestep. The transient numerical simulation, to get the time traces of the state variables, will be started in the Octave software using the ode23 solver with the following statement. The time to simulate is 0.03 s, and both initial values are 0.

```
[T,D] = ode23 (@Boost, [ 0.0 : 1 e-5:3e -2] , [0 0] );
```

D. Simulation Results

The resulting traces of the two state variables are presented in Figs. 10 and 11 for three different values of the duty cycle of the 31 kHz PWM signal.

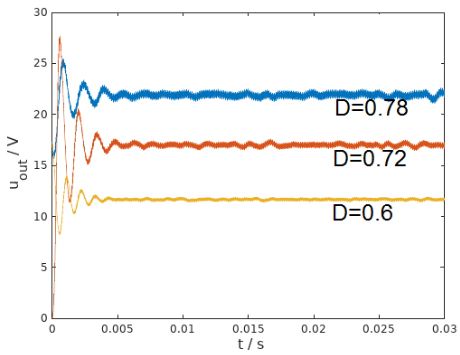

Fig. 10. Time traces of the output voltage for different duty cycles.

It can be seen that after some transient effects, depending on the initial values of the simulation, both state variables reach a (relative) steady state if computed using the numerical solver of Octave and also if the inductor is assumed to be dependent on its own current. Both signals can be described by a mean value with an additional ripple.

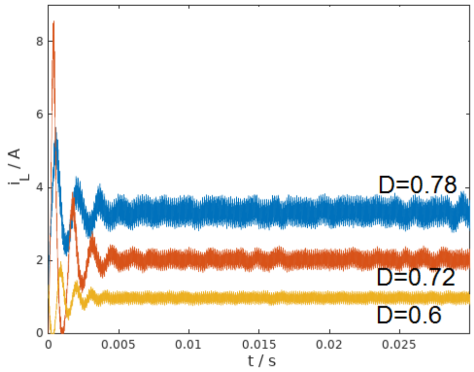

Fig. 11. Time traces of the inductor current for different duty cycles.

The most interesting aspect of the simulation results is the increase in the ripple current of the inductor with higher duty cycles.

The amplitude of the ripple current of the shown boost converter is dependent on the duty cycle as well as on the inductance value, as presented in Equation 11.

$$\Delta i_L = \frac{U_{in} \cdot T_{on}}{L} = \frac{U_{in} \cdot D}{f_{pwm} \cdot L} \tag{11}$$

The ripple current at a duty cycle of 0.78 is around 0.9 A, whereas the ripple current at a duty cycle of 0.6 is approximately 0.4 A. Following Equation 11, the increase in the ripple current due to the increase in the duty cycle from 0.6 to 0.78 would be 30%, if the inductor exhibits no saturation and stays at a constant value. Due to the consideration of the inductor saturation in the simulation, the increase in the current ripple is much higher compared with that at the two different duty cycles. They show an increase of approximately:

$$\frac{\Delta i_L(D = 0.78)}{\Delta i_L(D = 0.6)} = \frac{0.9\ A}{0.4\ A} = 225\%\,. \tag{12}$$

To check whether this result is in accordance with the saturation behavior, we included in the computational model, we compare the result of the numerical simulation to the increase in the ripple we can expect by changing the duty cycle from 0.6 to 0.78. At these two duty cycles, the average inductor current increases from 1 to 3.3 A. This increase in the current forces the choke to operate at a lower inductance according to Fig. 6 and Equation 7. The precise drop of the induction at the different mean currents is presented in Table I.

TABLE I: SIMULATION RESULTS AND SATURATION AT DIFFERENT DUTY CYCLES

D	I_{mean}	$L(I_{mean})$	ΔI
0.6	1 A	246 µH	0.4 A
0.78	3.3 A	141 µH	0.9 A

Considering both, the increase in the duty cycle and the decrease in the inductance due to its saturation, we get an analytically calculated increase in the duty cycle of

$$\frac{0.78 \cdot 246\ \mu H}{0.6\ \cdot 141\ \mu H} = 227\% \tag{13}$$

This value is close to the 225% increase in the ripple current that we derive in the simulation results and calculate in Equation 12. This good accordance of the analytically calculated and numerically simulated increase in the ripple current shows that the modeled saturation behavior is correctly considered in the numerical computation.

E. Comparison to a Simulation with Constant Inductor

If the inductivity in the simulation of the boost converter is kept constant to the on load value of 292 µH, the average output voltage ends up to be the same as that for the simulation with the saturating choke. The average value only depends on the turn-on time of the transistor.

$$U_{out} = U_{in} \cdot \frac{T}{T - t_{on}} \tag{14}$$

If the load (R_L=30 Ω) does not change, the average current will stay constant.

The drop of the inductance is again visible if we check the ripple height of the inductor current, which is shown in the following figure for a duty cycle of D=0.72.

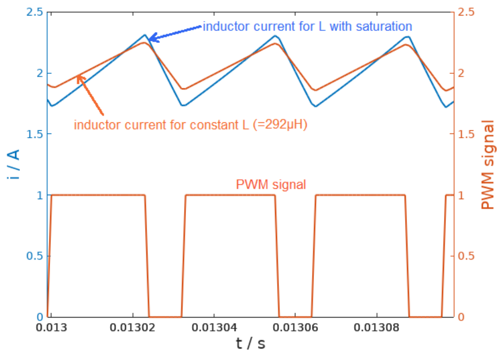

Fig. 12. Boost converter simulation results with and without considering choke saturation.

Figure 12 presents again the effect of the considered saturation. While both average currents are 2 A at a duty cycle of D=0.72, the ripple is significantly higher for the simulation, which considers the saturation of the choke. This is due to the drop of the inductance down to 193 µH, which follows the real behavior of the choke (this value is derived from Fig. 6 or calculated using Equation 7).

TABLE II: SIMULATION RESULTS: SATURATION VS. CONSTANT

	D	$L(@I_{mean} = 2\ A)$	ΔI
L=292 µH=constant	0.72	292 µH	0.4 A
L=L(i)=saturating	0.72	193 µH	0.6 A

The values in Table II are derived by simulation and relate to Fig. 12. They show again that the saturation has a direct influence on the simulation results, as the ripple current is significantly higher if saturation is taken into account, using the proposed method. The ripple height is inversely proportional to the inductivity, as expected considering Equation 11.

IX. CONCLUSION

In this paper, we presented a simple way to model a choke as a current-dependent device and demonstrated how to integrate this saturation behavior into a computational description in the form of a software function, which could be easily integrated in SPICE-like circuit simulators. On a simple test circuit, we showed that the same saturation behavior can be either inserted as a continuous function based on the core manufacturer's information or based on measurements of an existing choke if the information (material, AL-value, turns) of the inductor is not available. It is obvious that the method does not restrict the user to any mathematical form. It is also independent of the core material of the choke.

Using the example of a boost converter, we pointed out that the consideration of the choke saturation has a direct impact on the simulation results, in this case on the height of the ripple of the inductor current signal.

The presented approach can help in making the model of the chokes in a numerical time–space simulation more exact. If this saturation model is combined with parasitic parameter of the choke, like wire resistance and capacitance, which are easier to include, it will lead to even better simulation results. Those advanced models will help the circuit developer consider real-life effects and their impacts on his/her circuit design.

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